Electronic Energy Gap in Ferromagnets Due to a Strong Internal Magnetic Field

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The internal field in $3d$ ferromagnets is sufficiently strong that the motion of conduction electrons is affected to the extent that Landau quantization effects can be important. At low temperature we consider the scattering of conduction electrons from magnetic moments which are antiparallel to the direction of spontaneous magnetization and find an energy gap exists in the electronic energy spectrum. The current-carrying gap state gives rise to a frequency-dependent transverse conductivity which is enormous in the zero-frequency limit, suggesting an alternative mechanism for the spontaneous Hall effect in ferromagnets.

It is evident from Krinchik's[1] analysis of the Hall effect in Fe, Co, and Ni at optical frequencies and from Fermi-surface studies[2,3] of ferromagnetic metals that the strong internal magnetic field in $3d$ ferromagnets should be taken into account in describing the motion of conduction electrons. We consider the situation where Landau quantization effects (due to the internal magnetic field) dominate over interband effects and take into account the scattering of conduction electrons from magnetic moments which are antiparallel to the direction of spontaneous magnetization (only a few moments are antiparallel at low temperature). Evaluation of the electronic energy spectrum shows that one of the resonant states (as a result of electron scattering) is separated by an energy gap from the continuum of states. The current-carrying gap state gives rise to a transverse conductivity which is enormous in the zero-frequency limit.

Consider the scattering of a conduction electron in one spin band from an antiparallel magnetic moment. $U(\vec{r}, t) = f\delta(\vec{r})\delta(t)$, the interaction, is obtained from the contact part of the s-d interaction between the conduction electrons of one spin band and the d electrons (the origin of the magnetic moments). (At low temperature most of the magnetic moments are parallel to the direction of spontaneous magnetization; interaction of conduction electrons with these moments may be taken into account through the band structure.) The probability amplitude (for a conduction electron) to go from state Φ_n to state Φ_m is given by

$$T_{nm} = \int d^4x_1\, \Phi_m^*(1)\, U(1)\, \Phi_n(1) + \int d^4x_1 \int d^4x_2\, \Phi_m^*(1)\, U(1)\, G_0^+(1,\ 2)\, U(2)\, \Phi_n(1) + \int d^4x_1 \int d^4x_2 \int d^4x_3 \times \Phi_m^*(1)\, U(1)\, G_0^+(1,\ 2)\, U(2)\, G_0^+(2,\ 3)\, \Phi_n(3) + \cdots$$

$$= f\,\Phi_m(0)\,\Phi_n(0)\{1 + fG_0^+(0,\ 0) + [fG_0^+(0,\ 0)]^2 + \cdots\}\ . \tag{1}$$

$G_0^+(1,\ 2)$ is the retarded single-particle Green's function in a uniform (although the internal field is highly nonuniform, it appears from the measurement[2] of de Haas–van Alphen oscillations in iron that one may assume a uniform magnetic field as far as the motion of conduction electrons is concerned) magnetic field and $G_0^+(0,\ 0)$ is

$$G_0^+(0,\ 0) = \frac{-i}{\rho} \int \frac{d\omega'}{2\pi}\, \frac{1}{\hbar\omega - \hbar\omega' + i\epsilon} \int dP_z \times \int d\overline{P}\, \frac{2\pi\overline{P}}{(2\pi)^2} \int_{-i\infty+\delta}^{+i\infty+\delta} \frac{ds}{2\pi i}\, \exp s\left(\hbar\omega' - \frac{\hbar^2 P_z^2}{2m}\right) \times \left[\frac{1}{\cosh(\hbar\omega_c s/2)}\, \exp\left(\frac{-\hbar\overline{P}^2}{m\omega_c}\, \tanh\frac{\hbar\omega_c s}{2}\right)\right]\ . \tag{2}$$

$\omega_c = eH/mc$ is the cyclotron frequency, H is the internal magnetic field [notation: $\vec{H}$ the magnetic field is in the direction of the z axis, $\vec{P} = (P_z,\ \overline{P}) =$ wave vector], and ρ is the particle density. Expanding $G_0^+(0,\ 0)$ over Landau eigenstates $[E_j(p_z) = \hbar\omega_c(j + \frac{1}{2}) + \hbar^2 p_z^2/2m]$, the frequency-dependent transition amplitude $T_{nm}(\omega)$ is

$$T_{nm}(\omega) = f\,\Phi_m(0)\,\Phi_n(0) \bigg/ \left(1 + \frac{f}{\rho}\, \frac{\omega_c m^{3/2}}{\hbar^2 2^{3/2}\pi} \times \sum_{j=0}^{\infty} \frac{1}{[(j + \frac{1}{2})\hbar\omega_c - \hbar\omega]^{1/2}}\right)\ . \tag{3}$$

Assuming an attractive potential ($f < 0$), it is evident from (3) that resonances occur in the transition amplitude at frequencies near $(j + \frac{1}{2})\omega_c$. The resonance with $j = 0$ is of significance: It is lower in energy than all other scattering and resonance states and is isolated in the sense that it is removed from the continuum of scattering states (P_z dependent states). The zero-point energy of the $j=0$ resonance is

$$E=\tfrac{1}{2}\hbar\omega_c - E_G ,$$

and E_G is the binding energy,

$$E_G=\frac{f^2}{\rho^2}\,\frac{\omega_c^2 m^3}{\hbar^4 8\pi^2}$$

or the energy gap. Bound pairlike structures are formed consisting of conduction electrons circling (in quantized cyclotronic motion) antiparallel magnetic moments.

A current-carrying state separated by an energy gap from the bulk of electronic states gives rise to the possibility of the existence of persistent currents analogous to superconductivity. Taking into account the fact that magnetic moments can move, we evaluate the frequency-dependent transverse conductivity $\sigma_T(\omega)$ [as derived from a density-matrix point of view with the help of the works of Bychkov[4] and of Kahn and Frederikse[5]], and find $\sigma_T(\omega)$ is enormous in the zero-frequency limit:

$$\sigma_T(\omega)\simeq\int d\epsilon\;\frac{df_0(\epsilon)}{d\epsilon}\left[\left[\Delta(\epsilon)+\hbar\omega\right]\left(1+\frac{E_G^{1/2}}{\Delta^{1/2}(\epsilon)}\right)^2\right]^{-1}$$

$$\simeq-\left[\frac{1}{\Delta(\zeta)+\hbar\omega}+\frac{df_0(-\Delta(\zeta))}{d\zeta}\;\ln\!\left(\frac{E_G\,\hbar\omega}{\zeta\,\Delta(\zeta)}\right)\right], \quad (4)$$

where

$$\Delta(\epsilon)=\left|\tfrac{1}{2}\hbar\omega_c-\epsilon\right|$$

and $f_0(\epsilon)$ is the Fermi function. When finite electron lifetimes are taken into account, an additional term arises in the transverse conductivity [as, for example, in Eq. (12.13) of the article by Kubo, Miyake, and Hashitsume[6]].

The existence of an electronic energy gap might explain the spontaneous Hall effect in certain ferromagnets. The effect was observed by Dheer,[7] who found that the dc transverse conductivity of iron whiskers jumps by four orders of magnitude [going to $\sigma_{xy}(\Omega\,\mathrm{cm})^{-1}\sim 10^7$] when the temperature is lowered below the transition temperature T_s (between 20 and 80 °K). (The explanation of the effect in iron by Fivaz[8] has recently been cast in doubt by Duff and Das.[9]) With the s-d interaction, $f=0.5$ eV, the internal magnetic field $H=20$ kG, the electron mass $m=9.11\times10^{31}$ kg, and $\rho=2.4\times10^{16}$ electrons/cm^3, we find the energy gap is

$$E_G=\frac{f^2}{\rho^2}\,\frac{\omega_c^2 m^3}{\hbar^4 8\pi^2}=10\ ^\circ\mathrm{K}\ .$$

The electron density is consistent with the thought that only electrons contingent to small portions of the Fermi surface are affected by Landau quantization.

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[1]G. S. Krinchik, J. Appl. Phys. 35, 1089 (1964).

[2]J. R. Anderson and A. V. Gold, Phys. Rev. Letters 10, 227 (1963).

[3]A. V. Gold, J. Appl. Phys. 39, 768 (1968).

[4]Yu. A. Bychkov, Zh. Eksperim. i Teor. Fiz. 39, 689 (1960) [Sov. Phys. JETP 12, 483 (1961)]; see in particular p. 490.

[5]A. H. Kahn and H. P. R. Frederikse, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic, New York, 1959), Vol. 9, pp. 257–291.

[6]R. Kubo, S. J. Miyake, and N. Hashitsume, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic, New York, 1965), Vol. 17, pp. 269–364.

[7]P. N. Dheer, Phys. Rev. 156, 637 (1967).

[8]R. C. Fivaz, J. Appl. Phys. 39, 1278 (1968).

[9]K. J. Duff and T. P. Das, Phys. Rev. B 3, 192 (1971).